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Percolation models of competition and monopolisation in the agricultural market

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▶ **Abstract.** The study was relevant due to the growing risks of agricultural market monopolisation in Ukraine, which necessitated a quantitative analysis of concentration processes using modern modeling tools. The purpose of the study was to build a model of monopoly formation in the agricultural market by applying a percolation approach to forecasting phase transitions in a competitive environment. A two-dimensional percolation model of the agricultural market was developed to simulate the capture of market segments by large formations and assess concentration dynamics. Numerical experiments (200×200 domain) showed that as the control parameter approaches the critical value $P^*=0.5945$, the correlation coefficient of rating-frequency diagrams fell sharply from 0.94-0.97 at $P=0.50$-0.58 to 0.55 at $P=0.59$, indicating a phase transition interpreted as monopoly cluster formation. Using Ukrainian agricultural market data for 2017-2023, the model identified a critical percolation threshold at $P^*=0.59$, accompanied by a decline in correlation coefficients from 0.96 to 0.55. A logarithmic relationship $W=-0.3839-0.153\ ln|P-P^*|$, $R^2=0.9821$ described the growth of dominant clusters. The number of agricultural enterprises declined from 40.7 to 30 thousand (-26%) and average land per enterprise increased from 490 to 576 ha, confirming the intensification of concentration processes and illustrating how geometric cluster behaviour mirrors real structural shifts in the sector, thereby strengthening the applied significance of the developed modelling approach and providing a quantitative framework for detecting early signs of market dominance, assessing systemic vulnerabilities, and interpreting concentration dynamics through the lens of phase-transition phenomena. The practical value of the study lies in enabling early identification of market monopolisation and critical transition points, thereby supporting more accurate forecasting of structural shifts and the development of effective antitrust and regulatory measures

▶ **Keywords:** clustering; phase transition; market concentration; fractal dimension; market asymmetry; monopoly risk assessment

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Introduction

The growing concentration of agricultural capital, the decrease in the number of enterprises that own or use agricultural land, and the increasing influence of large agricultural formations on the Ukrainian market necessitated the application of the latest quantitative approaches to analyse and model monopolisation processes in agriculture. In the context of the modern globalised economy, the market operated as an open stochastic system prone to phase transitions and the formation of clusters with a high concentration of participants. In the agricultural sector, despite its traditional resistance to classical mechanisms of monopolisation, large corporate structures increasingly captured individual market segments, which was accompanied by a measurable decline in competition. M. Conyon *et al.* (2023) emphasised that consolidation processes in related industries often intensified due to nonlinear interactions and the accumulation of market power. These tendencies necessitated a quantitative assessment of monopolisation dynamics that accounted for the complexity, non-linearity, and multifactorial nature of interactions between market agents. Empirical studies in the agricultural economics literature also confirmed the growing structural asymmetries and the emergence of concentration clusters. P. Maranzano *et al.* (2025) demonstrated that farmland production in the European Union had experienced significant consolidation with a decline in the number of small farms and an increase in average operational trends that parallel the structural transformations observed in Ukraine. R. Cerqueti *et al.* (2025) employed a spatially-clustered spatial autoregressive model to identify regional "hotspots" of agricultural market concentration across Europe, revealing spatial patterns analogous to cluster formation in complex systems.

In addition to structural shifts driven by market forces, policy-induced dynamics also contributed to concentration. Researchers A. Mozdzen *et al.* (2024), using a Bayesian nonparametric partial clustering approach, showed that agricultural subsidies and institutional incentives often have heterogeneous effects and may reinforce consolidation tendencies in certain regions. E. Moretti *et al.* (2025) demonstrated through a spatial ecological-economic framework that farm size itself became a critical determinant of behavioural, ecological, and productivity characteristics, providing further theoretical justification for modelling agricultural markets as nonlinear systems with size-dependent dynamics. Economic studies, particularly within an interdisciplinary framework, increasingly integrates the tools of complex systems physics to explain economic phenomena. Percolation theory, phase-transition concepts, agent-based modelling, and mathematical ecology were widely applied to analyse transformations in competitive environments. R. Lucas (2022) demonstrated that network effects may induce nonequilibrium phase transitions in competitive markets, where monopolisation emerged as a consequence of symmetry breaking in conditions of unstable equilibrium. J.P. Nadal *et al.* (2003), using an agent-based computational economics (ACE) approach, revealed analogies between monopolistic pricing dynamics and first-order phase transitions in statistical physics. At the same time, the methodological versatility of percolation models was illustrated by their application in other scientific domains: W. Wang *et al.* (2023) modelled extraction and diffusion processes in pharmaceutical systems, demonstrating the potential of percolation-based approaches for studying the behaviour of complex multicomponent structures.

The study aimed to formalise and quantitatively model the processes of monopolisation in the agricultural market as cluster formation within a complex stochastic system, utilising methods from percolation theory and statistical physics tools. The study addressed the objectives: 1) to adapt percolation theory for modelling competitive market structures; 2) to identify critical thresholds of market concentration; 3) to assess socio-economic implications of monopolisation dynamics in agrarian sector. The scientific novelty consisted in applying percolation theory to quantify monopolisation processes in the agricultural market by identifying critical phase-transition thresholds and interpreting rating-frequency diagrams as indicators of emerging monopoly clusters within a stochastic competitive system.

Materials and Methods

Conceptual basis of modelling

The methodology was based on the concept of geometric phase transitions in complex stochastic systems, which was widely used in statistical physics, mathematical ecology, and complex network theory. At the first stage of the study, a hypothesis was formed about the similarity between market monopolisation processes and phase transitions in percolation systems. All types of competitive processes in science were characterised by an uneven distribution of results among participants, which can be generalised in the form of the parameter Q_i. In the context of this study, the parameter Q_i corresponded to the market share of an agricultural enterprise, which served as economic analogue of the cluster size in the percolation model. It has been established that in different types of systems, rating-frequency diagrams form patterns that were described by a semi-logarithmic function:

$$Q_i = a - b^* ln(i), \tag{1}$$

where Q_i – "achievements" of the i-th participant in the competitive process, i – participant number in the ranking, a, b – constants of the given competitive process, $ln(i)$ – natural logarithm of the ranking number i.

So, it was possible to construct quantitative models of such processes with high reliability. Particular attention has been given to geometric phase transitions, which were rigorously formalised within percolation theory as a mathematical framework that described how local interactions between elements of a system led to the formation of connected clusters and identified the critical threshold, at which a large-scale spanning structure emerges. This theoretical foundation made it possible to interpret the onset of monopolistic dominance in agrarian market environments as an analogue of a percolation-driven phase transition.

Building a two-component percolation model

In the second stage, a basic two-component percolation model was implemented, which described the interaction of two components – A (active market participants) and B (passive or displaced entities). In this model, a given area G with fractal dimension D (which may take integer or fractional values) was assumed to contain a mixture of components A and B. The area G is partitioned into N cells, each of which may be occupied by component A with probability P or by component B with probability (1-P) in accordance with the specified conditions:

$$\frac{N_A + N_B}{N} = 1. \tag{2}$$

The parameter L denoted the characteristic size of the region G, then:

$$L^D = N, \text{ thus } L = N^{1/D}, \tag{3}$$

where D – the fractal dimension of the space. When the control parameter was changed, the probability P of filling the area G with the component A was changed:

$$P \in [0; P_*]. \tag{4}$$

The probability of the appearance of a connecting cluster on the domain G can be found as:

$$W = \frac{1}{1+exp[\lambda_L(P-P_*)]}, \tag{5}$$

where W – denoted the probability of the emergence of a system-spanning (connecting) cluster, λ_L – a scaling parameter controlling the steepness of the percolation transition, P_* – the percolation threshold. As the control parameter P approaches the percolation threshold P_* from below, the probability of the emergence of a system-spanning (connecting) cluster tends to unity:

$$lim_{P \to P_*} W \Longrightarrow 1. \tag{6}$$

Equation (6) described the critical behaviour of the system near the percolation threshold, where the probability of forming a system-spanning cluster rapidly increased. From the point of view of the theory of competitive processes, this meant the formation of a connecting cluster, or the monopolisation of a part or all of the region G. It has been demonstrated that the percolation threshold primarily depended on the fractal dimension of the region G (strong dependence) and weakly depended on the system size L and the way the region G was divided into elementary cells. In the case of a Cartesian partitioning of the domain G, the percolation threshold was described by the Cartesian approximation proposed in percolation and multifractal modelling studies (Grabar & Kubrak, 2025):

$$P_* = 1 - ln\frac{D+1}{2}, \tag{7}$$

where D – the spatial (topological) dimension of the domain G, which characterised the dimensionality of the Cartesian partitioning of the system into elementary cells.

In the statistical drawing of components A and B on the domain G, certain combinations of the same name components occurred along the vertices of elementary "cube-cells" and along the sides (planes). In the percolation problem, contact along the vertices (points) does not lead to the formation of a connecting cluster, but contact along the sides or planes does.

Implementation of the computational experiment

For numerical modelling, the proprietary software package included two modules: PERCOL and PERCOL-statistic, which ensured both the generation of percolation fields and the analytical processing of simulation outputs. PERCOL – set a statistical draw according to the specified control parameter $P(A)$ on the domain G with dimensions $a_x a_y a_z$, where each a_j was [0…500], visualised the generated area and the clusters formed within it, automatically colours clusters, and determined the presence (YES) or absence (NO) of a spanning cluster connecting the boundaries of the domain. The module also generated a test report for each realisation. To extend these basic functions, PERCOL additionally provided standardised generation of multiple stochastic realisations, supported visualisation of cluster structures for different values of the control parameter P, and enabled automated detection of the transition from fragmented to connected configurations. This allowed interpreting the occurrence of a spanning cluster as an analogue of market monopoly formation within the simulated competitive environment. PERCOL-statistic performed the statistical processing of outputs, including ranking clusters by size, constructing rating-frequency diagrams, calculating correlation coefficients, and approximating the empirical distributions. Beyond these functions, the module aggregated the results from repeated realisations, computed averaged indicators to ensure the stability of quantitative patterns, and identified deviations in the correlation structure that signalled proximity to a geometric phase transition. These analytical capabilities enabled the diagnosis of threshold phenomena and the evaluation of monopolisation probability in a reproducible and formalised manner.

Model expansion:
Three- and polycomponent percolation

To study multifactorial processes of the agricultural market, the following has been implemented: three-component model, in which three types of participants were introduced (small, medium, and large enterprises) with probabilities P_a, P_b, P_c that satisfy the normalisation condition:

$$P_a, P_b, P_c + P_c = 1. \tag{8}$$

For convenience, as a special case, automodelling conditions were provided:

$$P_b = \lambda\, P_a\, P_c = \lambda\, P_b = \lambda^2\, P_a, \tag{9}$$

where λ – a scaling parameter describing the intensity of competitive advantage transfer between successive states, determining the relative weights of P_a, P_b, P_c.

Then the condition of normalisation:

$$P_a(1 + \lambda + \lambda^2) = 1. \tag{10}$$

Multicomponent model (for $n \geq 3$ components), which allowed taking into account more complex scenarios of market interaction. The procedures for generating, visualising, and statistical processing were in all cases similar to the two-component model, which ensured standardisation of the computational experiment.

Indicators for assessing market concentration and monopolisation

The approach was based on the following indicators:

1) concentration index (CR), defined as the sum of the shares of the three largest enterprises in the total market volume of homogeneous goods in percent:

$$\mathrm{CR}_{3} = \sum_{i=1}^{3} K_i, \tag{11}$$

where *K* – the share of the *i*-th enterprise's products in the industry, %. If the concentration index was close to 100%, the market was characterised by a high level of monopolisation;

2) Linda index was applied in EU competition analysis to identify dominance thresholds depending on the number of leading firms and was mainly used as a structural diagnostic indicator rather than a strict numerical threshold;

3) Herfindahl-Hirschman Index (IHH), used as a benchmark for determining the possibility of mergers and acquisitions. It was defined as the sum of the squares of the shares of production of the main enterprises producing products (services) in a certain industry (sphere of economic activity).

$$\mathrm{IHH} = \sum_{i=1}^{n} k_i^2, \tag{12}$$

where *k* – the share of production of the *i*-th enterprise in the industry, %; *n* – the number of enterprises operating in the relevant market.

Verification of results

To verify the adequacy of the model, the modelling results were compared with empirical data for the Ukrainian market over the period 2017-2023. Attention was paid to comparing the change in the correlation coefficients of the rating-frequency diagrams with the transition of the system, in such a way that a decrease in correlation serves as an indicator of a phase transition, which was interpreted as an analogue of the monopolisation process. The percolation model was constructed on the basis of aggregated empirical data characterising the structural dynamics of the Ukrainian agricultural market for the period 2017-2023. In particular, the model parameters were calibrated using official statistical indicators, including: the number of agricultural enterprises operating in Ukraine; the number of enterprises owning or using agricultural land; the total area of agricultural land; and the average agricultural land area per enterprise. These indicators were obtained from the statistical yearbooks and sectoral datasets of the State Statistics Service of Ukraine (2024), which served as the empirical basis for validating the simulated clustering and monopolisation dynamics.

Results and Discussion

Results of computational percolation experiments for agrarian market clustering

Numerical simulations of a two-component, two-dimensional percolation model was applied to the agricultural market, illustrating the evolution of cluster structures and the corresponding rating-frequency diagrams for different values of the control parameter *P*, which reflected the transition from a fragmented competitive state to a connected (monopolised) configuration. The use of rating-frequency diagrams as a diagnostic tool aligns with graphical modelling approaches applied in other complex systems, where visual structures were employed to reveal latent dominance patterns and influence mechanisms (Savchuk, 2018). To achieve the stated research objective, computer modelling of the kinetics of cluster formation in the agricultural market was carried out using the percolation approach. The most significant results in the theory of critical phenomena and phase transitions were obtained in the two-component percolation, which was due to the simplicity of implementation, the availability of visualisation and analytical processing of the results, since there was single control parameter . In turn, of all the possible values of fractal dimension of the region *G*, metric spaces are most often used: $D = 1$; $D = 2$; $D = 3$. Then, the value of the percolation threshold according to the Cartesian approximation gave a result that corresponded with an error of less than 1% to theoretical and numerical results reported in percolation theory for two-dimensional Cartesian lattices (Feder *et al.*, 2022; Shevchuk *et al.*, 2022):

$$\begin{gathered} P_{*/D=1} = 1 - ln\frac{1+1}{2} = 1; \\ P_{*/D=2} = 1 - ln\frac{2+1}{2} = 0.5945; \\ P_{*/D=3} = 1 - ln\frac{3+1}{2} = 0.3068. \end{gathered} \tag{13}$$

The most commonly used models were of domain G: "strips" $a \times B$ (*a* is 1…B), "squares" B x B (B is 2…500), "sandwiches" $B \times B \times a$ (*a* is 1…B). In the present study, the domain *G* was represented by three geometrical configurations. The "strips" correspond to quasi-one-dimensional lattices of size $a \times B$, modelling elongated systems. The "squares" represent classical two-dimensional lattices of size $B \times B$. The "sandwiches" denote quasi-two-dimensional layered systems of size $B \times B \times a$, where *a* is the thickness parameter. These configurations allow analysis of finite-size effects and the influence of dimensionality on stochastic percolation behavior. The results of computer modelling of a two-component ($A + B$) two-dimensional ($D = 2$) percolation process for the region $B \times B$ are presented. The study revealed that for values of $B > 100$, the character of the rating-frequency diagrams does not change significantly. The value of the control parameter *P* was varied over the range [0…1]. One of the most interesting was the range of these values:

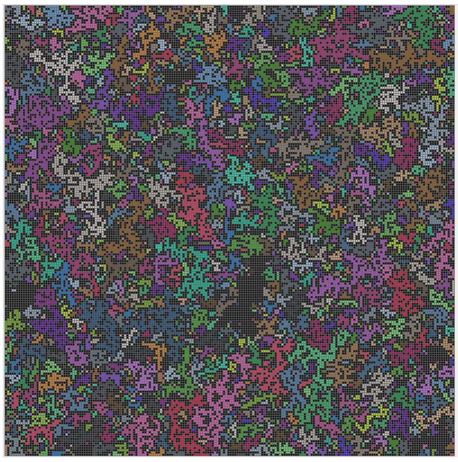
$$P \text{ is } [0.85\ldots\ 1.1]\ P_*, \tag{14}$$

where with a high correlation coefficient $R^2_{1/1} > 0.95$, rating-frequency diagrams were described by semilogarithmic dependencies of type (1). Parameters:

$$P > 0.97\, P^*, \tag{15}$$

where corresponding to the region of geometric phase transition, or the capture (monopoly) of one or more giant clusters of the region *G*, the value of the correlation coefficient in the semilogarithmic model of rating-frequency diagrams dropped sharply.

The results of modelling the formation of clusters were obtained for a range of control parameter values $P \in [0.50\text{-}0.59]$. To illustrate the structural evolution of the percolation field without redundant graphical repetition, four representative regimes were selected for detailed presentation: $P = 0.50$, $P = 0.54$, $P = 0.58$, and $P = 0.59$. For a two-dimensional model ($D = 2$), according to (1), the percolation threshold was $P^* = 0.5945$. Computer modelling was performed using the author's software products PERCOL and PERCOL-statistic. Although the capabilities of percolation-based modelling frameworks may be extended to larger domains and more complex configurations (Grabar & Kubrak, 2025; Grabar & Kilnitska, 2025), a reduced lattice size $Q = 200\text{x}200$ was sufficient to capture critical phase-transition effects. From a modelling perspective, the separation of simulation and analytical modules implemented in the PERCOL and PERCOL-statistic software corresponded to structured system design approaches that combined conceptual and formal representations, as proposed by M. Fu *et al.* (2018) for complex operational modelling. The software product PERCOL-statistic enabled the determination of cluster sizes and the construction of their rating-frequency diagrams. The obtained rating-frequency diagrams were best approximated in semi-logarithmic coordinates, yielding the highest correlation coefficients. Analysis of the graphs showed that the onset of a phase transition dramatically changed the kinetics of the rating-frequency diagrams for $P = 0.59$, accompanied by a sharp dropped in the correlation coefficient from 0.9-0.97 to a critically low value of 0.55. This may serve as an additional quantitative criterion for identifying a phase transition, analogous to the emergence of a monopoly. The results of the statistical modelling were analysed for 40,000 market participants. Intermediate configurations within $P \in [0.51\text{-}0.57]$ demonstrated gradual quantitative cluster growth without qualitative topological transformation and therefore were not presented separately. Figure 1 illustrated the cluster configuration of the percolation field at $P = 0.50$, corresponding to a fragmented competitive market state.



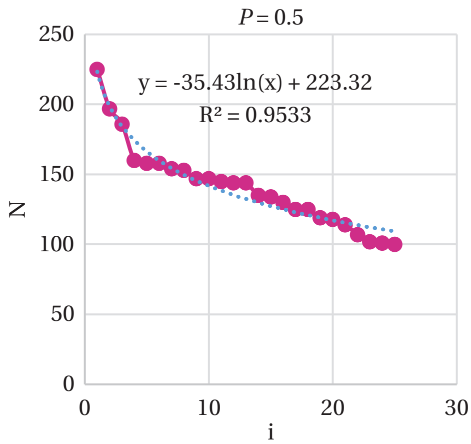


Figure 1. Model of cluster formation with the parameter $P = 0.5$

Notes: R^2 – the coefficient of determination, which characterised the goodness of fit of the logarithmic approximation to the simulation data and indicated the share of variance in the dependent variable explained by the model

Source: State Statistics Service of Ukraine (2024)

At $P = 0.50$, the percolation field was dominated by small, isolated clusters, and no system-spanning cluster was observed. This configuration corresponded to a decentralised market structure with a high level of competition and the absence of dominant market players. The spatial configuration at $P = 0.50$ demonstrated a highly fragmented topology characterised by numerous small clusters distributed relatively uniformly across the domain. The absence of large connected components indicated limited interaction between dominant agrarian market participants. The corresponding rating-frequency diagram preserved a stable semi-logarithmic pattern with a high coefficient of determination ($R^2 = 0.95$), confirming structural equilibrium. This regime reflects a competitive market state where resource distribution remains dispersed. A moderate increase in the control parameter within the range $P \in [0.51\text{-}0.53]$ led to gradual cluster enlargement and partial merging of neighbouring structures. However, the system remained below the percolation threshold, and no system-spanning cluster emerges. The rating-frequency diagrams within this interval maintain structural stability in semi-logarithmic approximation ($R^2 > 0.95$), indicating preservation of competitive balance. These intermediate configurations reflected quantitative growth without qualitative topological transformation. Figure 2 illustrated the cluster configuration at $P = 0.54$, where the enlargement of clusters became structurally pronounced and the first signs of large-scale aggregation emerge.

At $P = 0.54$, cluster enlargement became visually evident, and the spatial structure of the system begins to reorganise. Although a system-spanning cluster was still absent, several medium-sized clusters expand and absorb neighbouring elements, leading to increasing structural asymmetry. The corresponding rating-frequency diagram continues to follow a semi-logarithmic distribution, with a high coefficient of determination ($R^2 = 0.97$), indicating that the system remained within a competitive regime. However, the growth of dominant clusters signalled the gradual accumulation of structural instability. As the control parameter increased further within the interval $P \in [0.55\text{-}0.57]$, cluster coalescence intensifies, and the size distribution became increasingly uneven. The largest clusters began to dominate spatially, although full connectivity across the domain has not yet been achieved. In this range, the semi-logarithmic approximation remained generally valid, but fluctuations in the correlation coefficient indicated the system's approach to a critical state. Figure 3 illustrated the percolation field at $P = 0.58$, corresponding to a pre-critical configuration. At this stage, several large clusters occupied a substantial portion of the domain, and the system approaches the percolation threshold P^*. The spatial structure exhibited pronounced heterogeneity, and the emergence of near-spanning formations indicated imminent phase transition. At this stage, the correlation structure became less stable, reflecting the system's proximity to the critical percolation threshold.

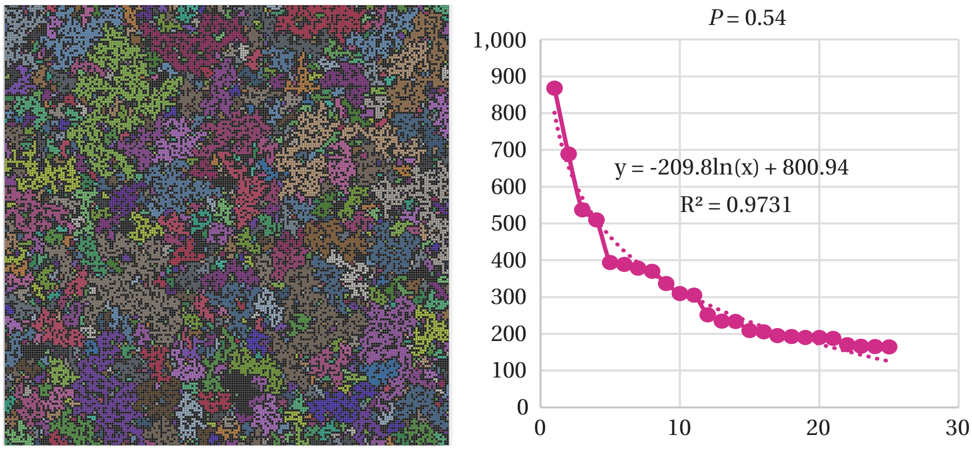


Figure 2. Model of cluster formation with parameter $P = 0.54$

Notes: R^2 – the coefficient of determination, which characterised the goodness of fit of the logarithmic approximation to the simulation data and indicated the share of variance in the dependent variable explained by the model

Source: State Statistics Service of Ukraine (2024)

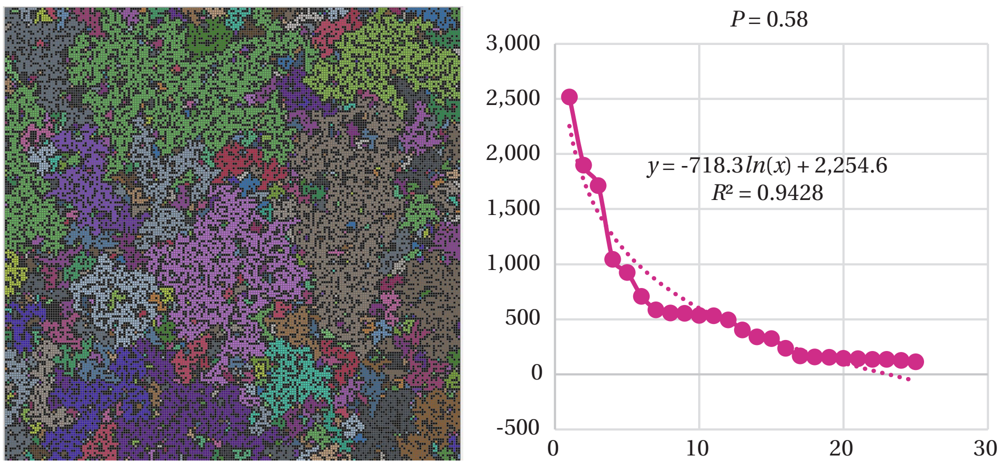


Figure 3. Model of cluster formation with parameter $P = 0.58$

Source: State Statistics Service of Ukraine (2024)

A further marginal increase of the control parameter to $P = 0.59$ results in a qualitative transformation of the system, marking the onset of a geometric phase transition at the percolation threshold. Figure 4 illustrated the formation of a system-spanning cluster, indicating that the percolation threshold has been effectively reached. In contrast to the pre-critical regime, connectivity now extended across the entire domain. Unlike the gradual structural evolution observed in previous regimes, this transition was abrupt and characterised by a fundamental reorganisation of spatial connectivity. At $P = 0.59$, the rating-frequency diagram deviates sharply from the previously stable semi-logarithmic behaviour. The coefficient of determination decreased dramatically from values in the range $R^2 = 0.94\text{-}0.97$ to approximately $R^2 = 0.55$, reflecting the breakdown of structural equilibrium. This abrupt declined serves as a quantitative signature of a geometric phase transition and can be interpreted as the emergence of monopolistic dominance within the modelled competitive system.

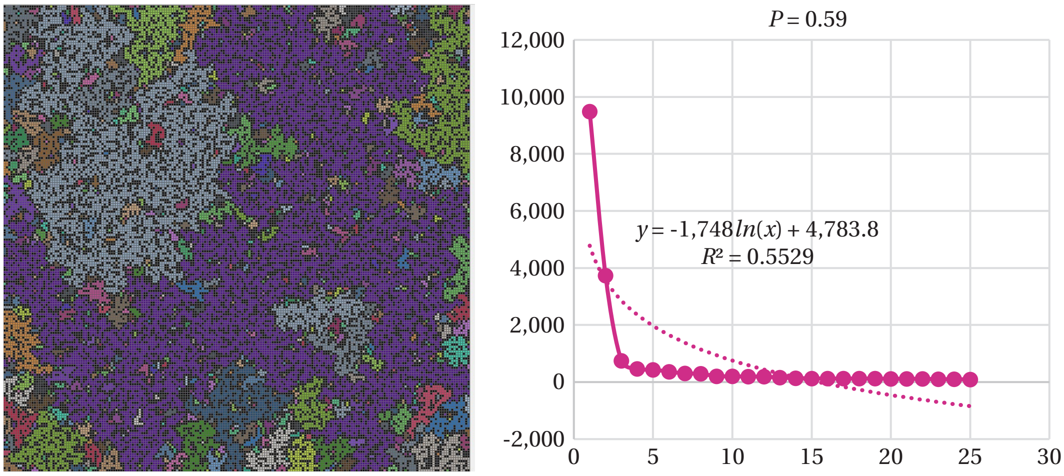


Figure 4. The model of cluster formation with the parameter $P = 0.59$
Source: State Statistics Service of Ukraine (2024)

The configuration shown in Figure 4 confirmed the formation of a system-spanning cluster, corresponding to a monopolised market structure, in which one dominant entity absorbed the majority of smaller participants. The rating-frequency diagram associated with this configuration (shown to the right of the spatial model) represents an ordered statistical distribution of cluster sizes for a system consisting of 40,000 elements. As the control parameter approaches the critical threshold P^*, the proportion of large clusters increased sharply, while smaller clusters were progressively absorbed, leading to rapid structural concentration. This reflected an accelerated process of structural stratification into "large" and "small" entities within the modelled competitive system. The dynamics of this transformation, observed as the parameter changes from $P = 0.50$ to $P = 0.59$, were quantitatively captured by the evolution of the correlation coefficient of the semi-logarithmic rating-frequency approximation. As illustrated in Figure 5, the correlation coefficient decreased from approximately 0.96 in the competitive regime to 0.55-0.60 near the phase transition point, providing a quantitative indicator of the breakdown of structural equilibrium.

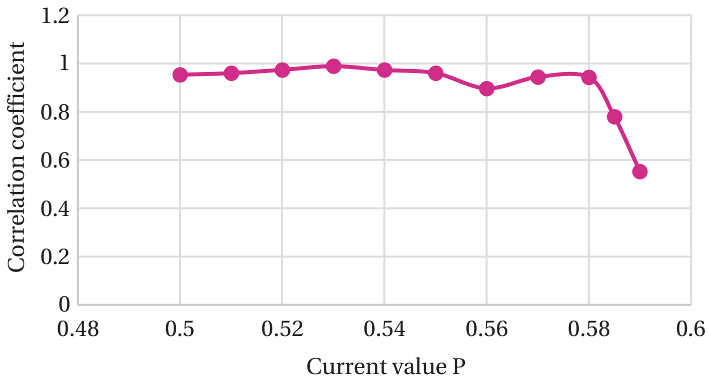


Figure 5. Dependence of the correlation coefficient of rating-frequency diagrams in semi-logarithmic coordinates, when approaching the percolation threshold P^*
Source: developed by the authors

Figure 6 presented the results of quantitative modelling of percolation-field coverage on a 200x200 (i.e., 40,000 participants) by the dominant clusters (row 1) and by the total of the three largest clusters (row 2), depending on the distance to the percolation threshold $|P - P^*|$. Each point of the graphs was obtained as the average value of ten independent simulation realisations performed in the "PERCOL statistician" environment. The obtained dependencies were accurately approximated by logarithmic models, as confirmed by high coefficients of determination. In particular, for the relative total share of the three leading clusters, the following relationship was obtained:

$$W = -0.3839 - 0.153\ ln|P - P^*|, \text{ with } R^2 = 0.9821. \quad (16)$$

The concentration index CR_3 was additionally computed for each modelled configuration as the cumulative share of the three largest clusters in the total system size (40,000 elements). The results indicated that CR_3 remained at a moderate level within the interval $P \in [0.50\text{-}0.57]$, corresponding to a competitive regime. However, as the control parameter approaches the critical threshold P^*, CR_3 increased sharply, indicating accelerated structural concentration. At $P = 0.59$, the CR_3 value reached a level consistent with high market concentration, confirming the transition to a monopolised structural regime. The Linda index was applied as a complementary structural diagnostic measure to assess asymmetry among leading clusters. Its dynamics near the percolation threshold further support the interpretation of the identified phase transition as a shift toward monopolisation.

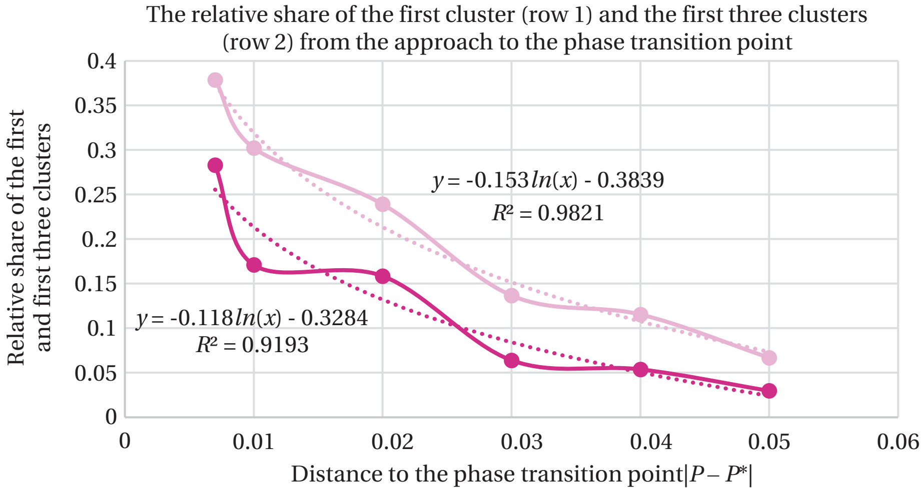


Figure 6. Results of quantitative modelling of the percolation field capture parameter 200x200 (40,000 participants), depending on the approach to the phase transition point
Source: developed by the authors

The dependencies shown in Figure 6 allowed the relationship between institutional antimonopoly constraints and the structural parameter $|P-P^*|$ to be conceptually assessed. The emergence of a system-spanning cluster may be interpreted not only as a structural transformation but also as an institutional threshold beyond, which regulatory mechanisms lose their effectiveness. As argued by D. Acemoglu & J.A. Robinson (2019), the balance between state capacity and societal constraints determined whether concentration processes remained controllable or evolve into extractive dominance. In this context, the identified percolation threshold may be interpreted as a structural boundary beyond, which institutional mechanisms became insufficient to restrain monopolisation dynamics.

Features of applying the percolation model of monopoly in the formation of agricultural holdings

The economy distinguished between different types of competition: perfect, imperfect, monopolistic (differentiated products), oligopoly (oligopsony), and monopoly. In practice, in real economic processes, the models function in conjunction, although one of them may be the basis. This was especially true of the level of manifestation – sectoral, territorial (local or regional), temporal, national. A monopoly in the market implied a dominant position of the producer, which was most often determined by the share (specific weight) of sales of the company's products in the total sales of homogeneous products of a particular industry. When studying the market structure (types of competition), quantitative methods were often used to assess the level of market concentration. The concentration of sellers reflected the relative size and number of enterprises, institutions, and organisations operating in a particular industry. The fewer business entities there were, the higher the level of concentration of sellers in the industry structure of the market. The greater the difference in size between businesses, the higher the level of concentration.

The legal criteria for enterprise size classification were defined in the Law of Ukraine No. 996-XIV (1999). According to this law, large enterprises were business entities with an average annual number of employees exceeding 250 and financial indicators (net revenue and/or balance sheet total) exceeding the thresholds established by national legislation and harmonised with EU accounting standards. Large enterprises tended to exert a dominant influence on market structure by contributing to increased concentration and centralisation of capital and production. On the one hand, this process was accompanied by economies of scale, while on the other hand, it may lead to intensified competition and the emergence of large corporate formations (cartels, syndicates, trusts, concerns, conglomerates) that occupied leading positions in specific industries or areas of economic activity. Key indicators of market monopolisation and enterprise size were: 1) share (specific weight) of sales of the enterprise's products in the total sales of homogeneous products of a certain industry; 2) share of employees in the enterprise in the total number of employees engaged in the production of these products; 3) share of the value of the enterprise's assets in the total value of assets of all business entities in the industry; 4) number of employees in the enterprise; 5) net income (revenue) from the sale of products (services); 6) cost of products (services) produced (Conyon *et al.*, 2023; Cerqueti *et al.*, 2025).

The relationship between monopolisation and the processes of concentration and centralisation was objective, i.e. functional. In O. Kilnitska's (2017) methodological approach, the assessment of an enterprise's monopoly power in the market was clarified. In particular, share of the company's products in the market of homogeneous goods (market niche in a particular industry or sphere of economic activity), which was determined as a percentage – *K*. Guided by the Law of Ukraine No. 2210-III (2001), a monopoly position was defined as a situation, in which an enterprise's shared in the market for a homogeneous product exceeds 35%. This criterion varied quantitatively across countries. For example, under Section 18 of the Act Against Restraints of Competition (1998), a single undertaking was generally presumed to be dominant if it held at least one third (33.3%) of the relevant market; dominance of two or three undertakings was presumed, when their combined market shared exceeds 50%, and of four or five undertakings, when it exceeded two thirds (66.7%). Based on the calculated indicator values, the level of agrarian market concentration was classified according to the criteria presented in Table 1.

Assessing the indicators of registered legal entities in the agricultural sector of Ukraine over the period 2017-2023, it can be concluded that overall market relations were becoming more competitive. Over this period, the number of registered agricultural enterprises has increased from 1,235,0 million in 2017 to 1,495,9 million in 2023. Thus, the number of business entities operating in the agricultural sector increased by an average of 37.2 thousand entities per year (Table 2).

Table 1. HHI thresholds applied for assessing concentration in the agricultural sector of Ukraine

Significance	Characteristics of market concentration	Implications for mergers and acquisitions
HHI < 1,000	Not concentrated	Mergers, acquisitions, and mergers are permitted
1,000 ≤ HHI ≤ 1,800	Moderately concentrated	If the HHI exceeds the 1,100 level, self-governing (management) bodies require verification of additional primary documents to permit business combinations
HHI > 1,800	Highly concentrated	Mergers, acquisitions, and mergers are prohibited

Source: developed by the authors

Table 2. Dynamics of registered legal entities in the agricultural sector of Ukraine by main indicators (2017-2023)

Indicator	2017	2018	2019	2020	2021	2022	2023	2023 to 2017	
								+/-	%
Total legal entities, thousand	1,235.0	1,298.4	1,350.6	1,395.4	1,437.0	1,464.9	1,495.9	260.9	121.1
of which: agriculture, forestry and fisheries	65,185	67,906	70,903	73,078	75,740	77,092	78,600	13,415	120.6
Share of agricultural, forestry and fisheries in the total number of legal entities, %	5.28	5.23	5.25	5.24	5.27	5.26	5.25	-0.03	99.4
Enterprises with agricultural land	40,735	40,333	38,523	36,277	39,301	29,631	29,991	-10,744	73.6
Share of enterprises with agricultural land in the total number of legal entities, agrarian land in the total number of agricultural, forestry and fishery enterprises, %	62.49	59.40	54.33	49.64	51.89	38.44	38.16	-24.33	61.1
Area of agricultural land, thousand ha	19,960.2	20,005.2	20,113.6	20,252.4	20,822.8	17,274.4	17,279.7	-2,680.5	86.6
Area of agricultural land on average per 1 agricultural enterprise, ha	490.0	496.0	522.1	558.3	529.8	583.0	576.2	86.2	117.6

Source: State Statistics Service of Ukraine (2024)

The data in Table 2 revealed several distinct trends in the agricultural sector during 2017-2023. First, the total number of enterprises engaged in agriculture, forestry, and fisheries increased from 65.2 thousand in 2017 to 78.6 thousand enterprises in 2023, reflecting a cumulative growth of 20.6% over the period under study. Their share in the total number of registered legal entities remained relatively stable at around 5.23-5.28%, indicating structural stability of the sector within the national economy. In contrast, the number of registered agricultural enterprises that owned or used agricultural land showed a downward trend during 2017-2023. In 2017, their number amounted to 40,735 thousand legal entities, whereas in 2023 it declined to 29,991 thousand enterprises, i.e., by 10,744 thousand entities over seven years, with the largest decrease recorded after February 2022. The share of enterprises with agricultural land in the total number of agricultural, forestry, and fisheries entities decreased from 62.49% in 2017 to 38.16% in 2023. These changes reflected not only human and business losses but also a contraction of production and land resources. Accordingly, the area of agricultural land owned by Ukrainian agricultural enterprises decreased from 19,960.2 thousand hectares in 2017 to 17,279.7 thousand hectares in 2023, i.e., by -2,680.5 thousand hectares. Monopolisation processes were closely linked to property relations. Therefore, when these processes were examined in agriculture, it became evident that, given the specific characteristics of this production sector, tendencies toward increasing concentration of agricultural land ownership have intensified. These trends indicated growing structural concentration in the Ukrainian agricultural sector. Against the background of a decrease in the number of agricultural enterprises engaged in land cultivation, the average agricultural land area per enterprise increased by 86.2 hectares over 2017-2023, from 490.0 hectares in 2017 to 576.2 hectares in 2023. During 2017-2023, according to the distribution of agricultural enterprises in Ukraine by the size of agricultural land, there was a reduction (through acquisition, merger, or liquidation) in the number of small and micro enterprises with land areas of up to 100 hectares, in favour of medium- and large-sized economic entities (Table 3).

Table 3. Dynamics of distribution of agricultural enterprises in Ukraine by the size of agricultural land, units

Size of agricultural land, ha	2017	2018	2019	2020	2021	2022	2023	2023 to 2017	
								+/-	%
Number of enterprises									
From 0.1 to 100.0	25,835	25,264	23,393	20,934	21,256	14,413	14,601	-11,234	56.5
From 100.1 to 1,000.0	10,023	10,277	10,389	10,605	12,599	10,670	10,857	834	108.3
More than 1,000.1	4,877	4,792	4,741	4,738	5,446	4,548	4,533	-344	92.9
Total	40,735	40,333	38,523	36,277	39,301	29,631	29,991	-10,744	73.6
Area of agricultural land, thousand hectares									
From 0.1 to 100.0	870.6	859.9	818.6	755.1	794.2	577.4	599.3	-271.3	68.8
From 100.1 to 1,000.0	3,688.5	3784	3,808.8	3,886	4,604.4	3,905.6	3,909.4	220.9	106.0
More than 1,000.1	15,401.1	15,361.3	15,486.2	15,611.3	15,424.2	12,791.4	12,771	-2,630.1	82.9
Total	19,960.2	20,005.2	20,113.6	20,252.4	20,822.8	17,274.4	17,279.7	-2,680.5	86.6

Source: State Statistics Service of Ukraine (2024)

The share of agricultural enterprises with land resources of up to 100 hectares decreased from 63.42% in 2017 to 48.68% in 2023. In contrast, the share of agricultural enterprises with an average land availability of 100.1 to 1,000 hectares increased from 24.61% in 2017 to 36.2% in 2023, and large enterprises (with an area of more than 1000.1 hectares) from 11.97% to 15.11%, respectively (Fig. 7).

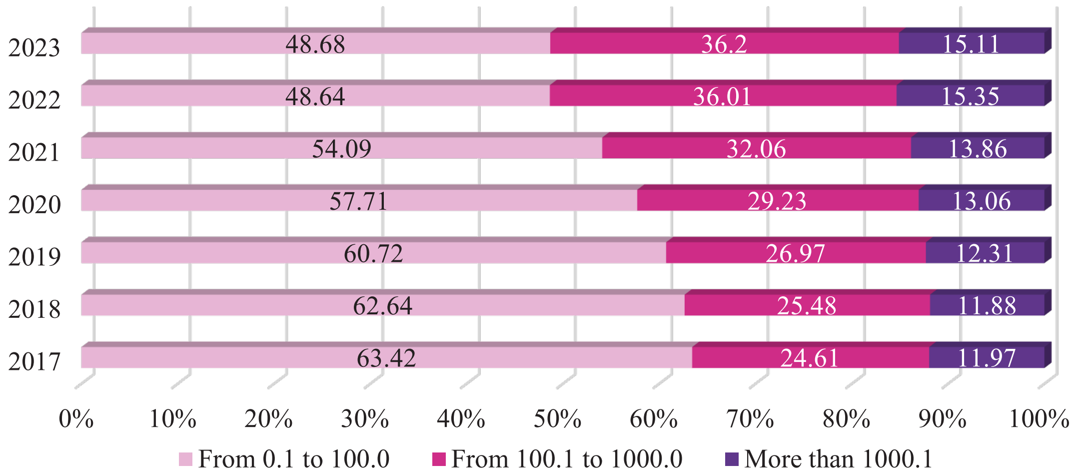


Figure 7. Structure of agricultural enterprises in Ukraine by size of agricultural land
Source: State Statistics Service of Ukraine (2024)

The redistribution of agricultural enterprises naturally affected the level of land use. While in 2017, 63.42% of enterprises had low land availability (up to 100 hectares per enterprise) and accounted for 4.36% of Ukraine's agricultural land, in 2023, 48.68% of these enterprises accounted for 3.47% of agricultural land. Large enterprises with the highest level of land availability (more than 1,000 hectares) increased their share in the structure of enterprises from 11.97% to 15.11% and, at the same time, controlled more than 73% of the country's agricultural land (Fig. 8).

The structural shifts illustrated in Figure 8 confirmed the tendency toward increasing concentration of land resources, which can be analytically interpreted within the framework of percolation-based modelling. The obtained results of quantitative modelling of clustering processes in the agricultural sector made it possible to interpret monopolisation not only as a legal or economic phenomenon, but also as a phase transition in a stochastic system, accompanied by the emergence of a connecting cluster, a dominant entity that covered a critical share of resources. Unlike traditional economic indices of concentration, such as the Herfindahl-Hirschman Index or concentration ratios, the percolation model allowed tracking the dynamics of monopoly structures during their formation, rather than merely recording existing market asymmetry. In particular, the Herfindahl-Hirschman Index and concentration ratios were ex post facto metrics, whereas the proposed model enabled identification of the critical zone, i.e., the moment, when the system loses competition as a mode of functioning.

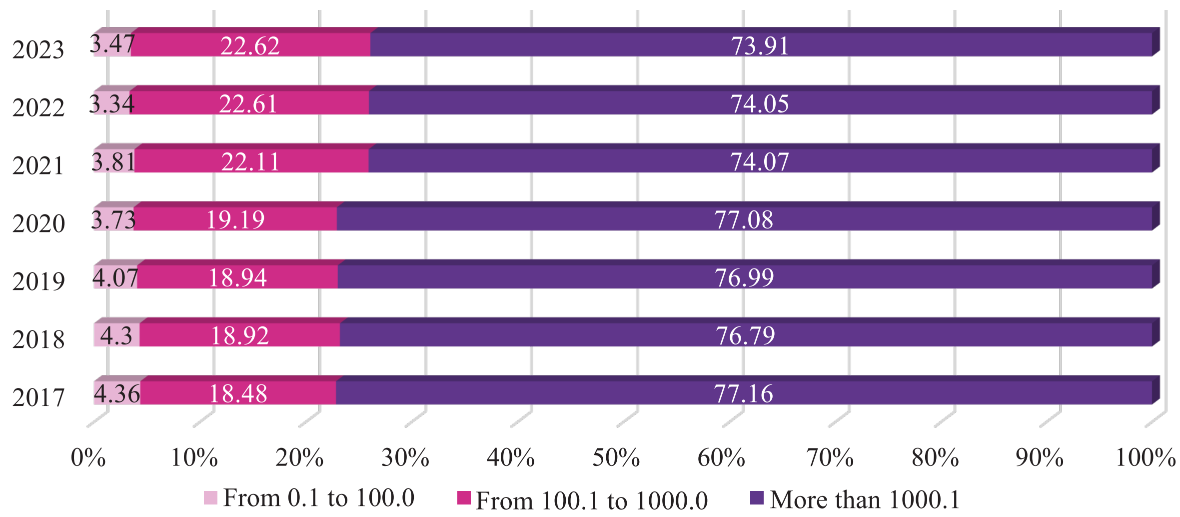


Figure 8. Structure of agricultural land owned by Ukrainian enterprises by size
Source: State Statistics Service of Ukraine (2024)

Contemporary economic research increasingly relied on interdisciplinary approaches that integrated methods of statistical physics, fractal analysis, and complex systems theory to study non-linear structural transformations. S. Donets *et al.* (2023) noted that fractal and percolation-based methods have proven effective for identifying clustering effects and critical thresholds in heterogeneous systems. E. Druzhinin *et al.* (2021) and D.L. Nguyen *et al.* (2023) emphasised that similar modelling approaches were actively applied in the analysis of distributed technical and network systems, including UAV swarms and decentralised control architectures, where cluster formation and phase-transition effects played a key role. These methodological advances provided a theoretical and computational foundation for applying percolation models to the analysis of market concentration and monopolisation processes in the agricultural sector. Recent advances in percolation theory and network-based modelling further demonstrated the applicability of phase-transition concepts to heterogeneous materials and distributed systems. T. Shevchuk *et al.* (2022) noted that percolation characteristics have been successfully employed to analyse structural connectivity and critical thresholds in complex media. These studies reinforced the relevance of percolation-based approaches for modelling the emergence of dominant structures in complex economic systems. I. Grabar & Y. Kubrak (2025) and I. Grabar & O. Kilnitska (2025) analysed percolation and fractal-based modelling tools for analysing clustering and phase-transition effects in complex systems. These methodological approaches provided a mathematical foundation for extending percolation-based models to the study of market concentration and monopolisation processes. Two-dimensional percolation model implemented in the software tools PERCOL and PERCOL-statistic enabled the analysis of conditions, under which connecting clusters-analogues of monopolistic formations-emerge. Furthermore, the study introduced rating-frequency diagrams as a quantitative diagnostic tool for identifying phase-transition signatures and determining critical thresholds of market concentration that signal an increased likelihood of monopolisation.

Conclusions

The agricultural market, despite its general structural resistance to monopolisation processes, demonstrated periodic manifestations of concentration that led to the absorption of small entities by large agricultural formations. Empirical analysis of the Ukrainian agricultural sector over the period 2017-2023 showed that the number of agricultural enterprises owning or using land decreased from 40,735 thousand to 29,991 thousand, while the average agricultural land area per enterprise increased from 490.0 to 576.2 hectares, indicating a significant intensification of land concentration. To quantitatively analyse and reproduce the kinetics of such processes, this study applied tools of percolation theory. The percolation model demonstrated its suitability as an approach to modelling the formation of monopoly structures in the market. The results indicated that the transition to a monopolised market structure occurs near $P = 0.59$, with a sharp decrease in structural stability indicators observed for the period 2017-2023. Numerical simulations revealed a critical percolation threshold at $P^* = 0.5945$, near which the correlation coefficient of rating-frequency diagrams sharply decreased from 0.94-0.97 to 0.55, indicating a geometric phase transition that corresponded to the emergence of monopoly dominance. It has been established that the capture of market segments by one or more large entities was physically analogous to phase transitions in statistical physics. The developed software tools PERCOL and PERCOL-statistic enabled visualisation and statistical diagnostics of clustering processes, providing a quantitative framework for early detection of monopolisation risks and supporting evidence-based antitrust regulation in the agricultural sector. Future research may focus on extending the percolation-based framework by incorporating dynamic institutional and policy factors, as well as applying the model to other sectors with heterogeneous spatial resource distribution.

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Conflict of Interest
None.

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Перколяційні моделі конкуренції та монополізації на аграрному ринку

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▶ **Анотація.** Дослідження є актуальним через зростаючі ризики монополізації аграрного ринку в Україні, що потребує кількісного аналізу процесів концентрації за допомогою сучасних інструментів моделювання. Метою дослідження було побудувати модель формування монополії на аграрному ринку, застосовуючи перколяційний підхід для прогнозування фазових переходів у конкурентному середовищі. Розроблено двовимірну перколяційну модель аграрного ринку для моделювання захоплення ринкових сегментів великими утвореннями та оцінки динаміки концентрації. Числові експерименти (область 200×200) показали, що з наближенням контрольного параметра до критичного значення $P^* = 0{,}5945$, коефіцієнт кореляції діаграм рейтингів і частот різко впав з 0,94-0,97 при $P = 0{,}50$-$0{,}58$ до 0,55 при $P = 0{,}59$, що вказує на фазовий перехід, що інтерпретується як формування монопольних кластерів. Використовуючи дані аграрного ринку України за 2017-2023 роки, модель виявила критичний поріг перколяції при $P^* = 0{,}59$, що супроводжувалося зниженням коефіцієнтів кореляції з 0,96 до 0,55. Логарифмічний зв'язок $W = -0{,}3839 - 0{,}153 \ln|P - P^*|$, $R^2 = 0{,}9821$ описує зростання домінуючих кластерів. Кількість аграрних підприємств знизилася з 40,7 до 30 тисяч (-26 %), а середня площа на одне підприємство збільшилася з 490 до 576 га, що підтверджує інтенсифікацію процесів концентрації та ілюструє, як геометрична поведінка кластерів відображає реальні структурні зрушення в секторі, тим самим зміцнюючи прикладне значення розробленого підходу до моделювання і надаючи кількісну основу для виявлення ранніх ознак монополізації ринку, оцінки системних вразливостей та інтерпретації динаміки концентрації через призму феноменів фазових переходів. Практична цінність дослідження полягає в можливості раннього виявлення монополізації ринку та критичних точок переходу, що сприяє більш точному прогнозуванню структурних змін і розробці ефективних антимонопольних та регуляторних заходів

▶ **Ключові слова:** кластеризація; фазовий перехід; концентрація ринку; фрактальна вимірність; асиметрія ринку; оцінка ризику монополії